ON THE HIGH ORDER MULTIPLICITY MOMENTS

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Abstract

The description of multiplicity distributions in terms of the ratios of cumulants to factorial moments is analyzed both for data and for the Monte Carlo generated events. For the PYTHIA generated events the moments are investigated for the restricted range of phase-space and for the jets reconstructed from single particle momenta. The results cast doubts on the validity of extended local parton-hadron duality and suggest the possibility of more effective experimental investigations concerning the origin of the observed structure in the dependence of moments on their order.

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1 Introduction

For decades the investigation of the moments of multiplicity distributions for high energy hadroproduction processes has been limited to the lowest three orders. It was generally believed that the experimental errors make the results for the fourth and higher cumulants meaningless. The investigations of higher normalized factorial moments in terms of intermittency [1] was possible by means of averaging the distributions over many bins in phase space, thus efficiently increasing the statistics. Still, even in this case the fourth and higher moments seemed to be determined to large extent by the values of the first three moments.

The breakthrough came with the analysis of Pavia group [2], who have shown that the analysis can be significantlly extended for a new kind of moments: the ratios of cumulants to the factorial moments

$$H_q = K_q/F_q.$$

Here the factorial moments are defined in the standard way

$$F_q = \sum_n \frac{n!}{(n-q)!} P(n),$$

and the cumulants may be calculated from a recursive formula

$$K_q = F_q - \sum_i \frac{(q-1)!}{(i-1)!(q-i)!} K_{q-i} F_i.$$

For the theoretical models it is sometimes easier to calculate moments from the generating function of the multiplicity distribution

$$g(z) = \sum_n z^n P(n).$$

The corresponding formulae read

$$F_q = \frac{d^q}{dz^q} g(z)|_{z=1}; \quad K_q = \frac{d^q}{dz^q} \log g(z)|_{z=1}.$$

If the values of the F_q moments of the order $q > 3$ are determined by the values of lower order moments, the values of the K_q moments are consistent with zero. In the Pavia group analysis the H_q moments were shown to be significantly different from zero for $q = 4,\ 5$ and 6, and possibly also for $q > 8$. The reason why the new moments of higher orders seem to have much smaller relative errors than the cumulants of the same order is the cancellation of some contributions to the overall errors. Fluctuations in the high multiplicity tail affect in a similar way the cumulants and factorial moments, and the resulting fluctuations of the values of their ratios are damped.

The behaviour of the moments found by the Pavia group initiated much interest because of their apparent compatibility with the predictions of perturbative QCD at the NLLA level [3],[4]. In both cases the dependence of moments on their order was found to be non-monotonical: after a minimum (with negative value) at $q = 5$, a hint of oscillations at higher q was seen. Later, however, some doubts appeared about the

origin of the observed structure. In particular, a simple cut in the smooth multiplicity distribution (e.g. of the negative binomial type) was shown to produce similar effects [5]. In this note we analyze the behaviour of the moments calculated from the PYTHIA generator [6] for the electron-positron annihilation at LEP-I energy and compare them, where possible, with the experimental L3 results [7], [8]. We include also some proposals for future investigations.

2 Data and Monte Carlo results for Z decays

To measure reliably higher multiplicity moments, one needs high statistics and negligible systematic errors. The LEP-I data, where millions of events have been collected by each experiment using detectors covering almost 4π solid angle with very high efficiency, seem to be ideal for this purpose. The results of L3 collaboration [7] were published recently (and compared with JETSET [9] and HERWIG [10] Monte Carlo results). In this section we remind these results and compare them with the moments calculated from PYTHIA. The comparison is only qualitative, as we do not use the L3 detector simulation program, which served to unfold the experimental multiplicity distribution [7]. Nevertheless, we will see that some surprising features of data and MC results are confirmed. We should note also that the overall event characteristics is properly described by PYTHIA: e.g., the values of the average charged multiplicity (above 18) and of the dispersion (about 5.9) are reproduced correctly.

The values of H_q moments calculated from the unfolded multiplicity distributions are reproduced in Fig.1a for the order $q = 4 \div 19$ together with the expectations of JETSET (the points for $q = 2, 3$ are never shown in such plots, because they lie much higher). It is obvious that JETSET results agree with data well (we do not reproduce here the unsuccessful comparison of data with HERWIG). However, it is equally obvious that the errors read out directly from the figures seem overestimated. The JETSET results (obtained without tuning the parameter values) never deviate from data by more than one standard deviation, and for the majority of moments agree with data within less than half of SD. This is statistically improbable and suggests that the values shown cannot be interpreted as simple uncorrelated statistical errors. Indeed, they are just the diagonal elements of the covariance matrix which is influenced by large bin-by-bin correlations.

The authors seem to recognize this problem, and they do not use directly the multiplicity distributions obtained from the data by an unfolding procedure. Instead they use the multiplicity distributions truncated in such a way, that multiplicities with relative error on $P(n)$ greater than 50% are rejected. This corresponds to the cut at highest multiplicities removing about 0.035% of events. With such a truncation the errors are visibly reduced and the clear structure appears both in data and MC results shown in Fig.1b: a minimum with negative values of H_q at $q = 5 \div 6$, and possible oscillations with maxima for q around $10 \div 11$ and $16 \div 17$.

In our opinion the lack of fluctuations above one standard deviation and the reduction of errors after the removal of some data suggest strongly that the error estimate may be misleading. Thus for our simulations using PYTHIA we do not calculate errors from standard statistical formulae (relating the errors of the q-th moment to some moments of

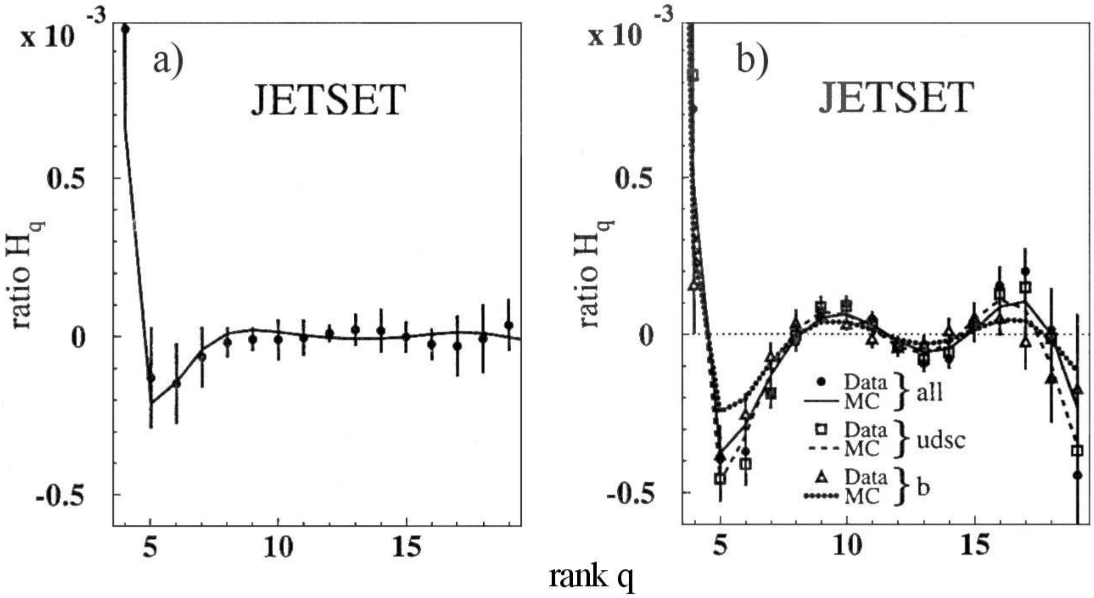

Figure 1: *The L3 results [7] for H_q moments calculated from full (a) and truncated (b) multiplicity distribution. Black points represent the data, and solid curve the events generated from JETSET. In Fig. 1b additional points and curves correspond to the separated udsc and b quark contributions.*

order $2q$), but use instead the uncertainty evaluated from the spread of results for different samples of independently generated events. We use the statistics, for which the errors are comparable to the size of symbols in the figures.

The results for samples of 4 million events (more than twice the experimental sample of L3) are shown in Fig. 2. We have checked that the choice of PYTHIA parameters (default values, or the values tuned to L3 data) does not influence significantly our results. We restricted our calculations to $q < 11$, as the results for higher q seem unstable even for such a huge statistics. We confirm the shallow minimum in the untruncated data, which becomes deeper after introducing the cut. Cutting off bigger part of the high multiplicity tail makes the minimum still more pronounced.

We checked that the minimum in untruncated data does not change significantly if we change the number of generated events between 2 and 4 million; thus it is not due to the "natural cut" resulting form the finite statistics. This confirms once more the results from many other investigations: the existence of minimum seems to be universal, but its shape depends strongly on the multiplicity cuts. If the statistics is low, such cuts are "naturally" present in data; for high statistics experiments they may result from detector deficiencies.

The origin of the presence of a structure in the dependence of the H_q moments on their order q for the PYTHIA events is not clear. We think that these effects are not due to the NNLO order QCD perturbative effects [3], [4], as the PYTHIA generator does not contain explicitly such components. A more plausible explanation is to relate the structure more generally to the multicomponent character of the production process [11]. An example of such "multicomponent expansion" for Z decays may be a separation of two-, three- and

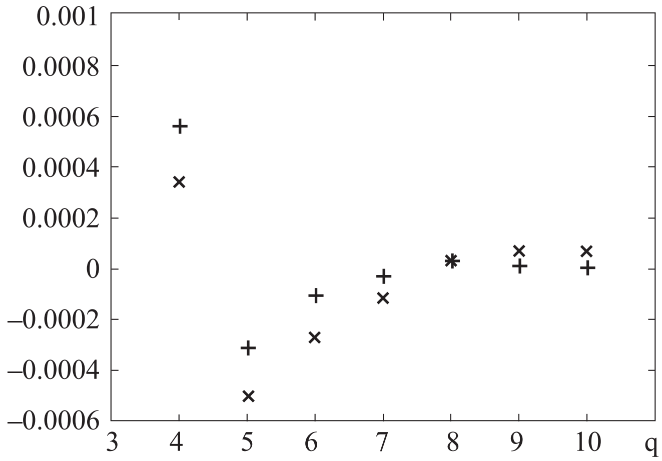

Figure 2: *The H_q moments calculated from PYTHIA events for L3 parameters . Crosses represent full distributions and x-s the distributions with the highest multiplicity tail (0.035% of cross - section) removed.*

multijet classes of events. It seems natural to assume a smooth (e.g. negative binomial - NBD) distribution to the two-jet events [12]. For such a distribution characterized by two parameters, $\overline{n}$ and k

$$P_n^{NBD}(\overline{n},k) = \frac{\Gamma(n+k)}{n!\Gamma(k)} \left(\frac{\overline{n}}{\overline{n}+k}\right)^n \left(\frac{k}{\overline{n}+k}\right)^k$$

one can easily calculate the factorial moments

$$F_q^{NBD}(\overline{n},k) = \left(\frac{\overline{n}}{k}\right)^q \frac{\Gamma(k+q)}{\Gamma(k)}.$$

The distributions for three- and multijet events should be also related to NBD. In a toy model, in which only two- and three-jet events are taken into account, and the parameters in this two classes are scaled in the $2:3$ ratio we have

$$P_n = \alpha P_n^{NBD}(\overline{n},k) + (1-\alpha) P_n^{NBD}(3\overline{n}/2, 3k/2).$$

Obviously, in such a toy model we get for the factorial moments

$$F_q = \alpha F_q^{NBD}(\overline{n},k) + (1-\alpha) F_q^{NBD}(3\overline{n}/2, 3k/2)$$

and from the formulae given in the Introduction one may easily write down the explicit formulae for the H_q moments of any order. We have calculated these moments for $q < 16$ for the wide range of model parameters. We found that for the values of $\overline{n}$ and k around 20 (as suggested by the experimental values of the average multiplicity and dispersion), and for $a \sim 0.85$ (corresponding to a 15% admixture of three-jet events, as suggested by data) a minimum for $q = 5$ (with a value around -0.0005) appears naturally. We checked also that a cut removing highest multiplicity events (at the level of 0.01%) enhances such a minimum. Thus any generator reproducing correctly the two- and three-jet components

may be expected to reproduce also qualitatively the observed structure in the dependence of H_q moments on their order q.

We conclude that the minimum in the dependence of moments H_q on their order q seems to occur naturally at $q = 5 \div 6$ for the hadron multiplicity distributions in the full phase-space. Its shape depends, however, on the details of the generating procedure. In particular, the cuts removing high multiplicity tail enhance the minimum quite strongly.

3 Higher moments in restricted parts of phase-space

The data and simulations discussed above concerned the full phase-space multiplicity distributions, unless limited by the experimental conditions. It is well known, however, that the multiplicity distributions in limited regions of phase-space depend significantly on the definition of the limits. For the early collider data [13] the successful negative binomial (NB) fits were found both for full available phase-space and for the intervals of CM rapidity. However, the $1/k$ parameter (equal to second normalized cumulant for NB distributions) increases significantly for a decreasing size of the rapidity interval. It was approximately described by the hypothesis of $\overline{n}/k$ scaling [14] or by the minimal model [15], in which the only significant correlations were those reflected by the multiplicity distributions for full phase-space. The data were not accurate enough to measure higher order cumulants and the statistics was insufficient to investigate small intervals (which was done later for intermittency effects using the averaging over many intervals [1]).

The LEP1 data have sufficient accuracy and statistics to investigate higher moments for various regions of phase-space. Thus we have performed calculations of such moments for the events generated with PYTHIA, defining phase-space regions by cuts in values of CM momenta. It should be interesting to see if the patterns revealed by such calculations will be confirmed by future data analysis.

We have generated samples of $4M$ events both for the default values of PYTHIA parameters and for the values used by L3 collaboration. We calculated the moments for orders $2 \div 10$ for regions defined by simple inequality for the values of the CM three-momenta p: $p < \epsilon n$, where $\epsilon = 0.2GeV$, and $n = 1 \div 10$.

Although the results for two choices of parameters are different, there is a common pattern in them; thus we show only the results for $L3$ parameters. The minimum (with negative values) in the dependence of moments H_q on their order q occurs for small phase space regions very late (at q around 8) and shifts gradually to smaller values of q for increasing range of CM momenta. However, even for the widest range of momenta investigated, the position and shape of minimum differs significantly from that for full phase-space. The results are shown in Fig.3a; for transparency we show only points for n =4,6,8 and 10.

In the previous chapter we have seen that the minimum for full phase-space was strongly enhanced by introducing cuts in the multiplicity distributions, although for the LEP1 data the minimum cannot be explained just by such a naturally occurring cut due to the finite statistics. Thus we repeated our calculations cutting off the highest multiplicities contributing 0.001 to the full cross-section. The results, shown in Fig. 3b, are quite surprising: now the position of minimum is practically independent on the size

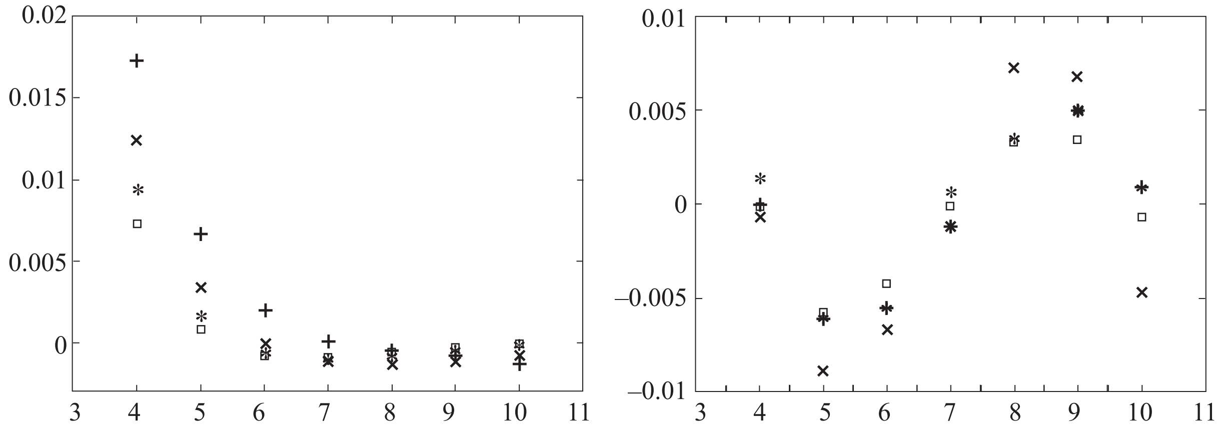

Figure 3: *The H_q moments calculated from PYTHIA events in limited parts of phase-space (see text) for L3 values of the parameters without (a) and with (b) a cut removing highest multiplicities. Crosses, x-s, stars and squares represent the results for n = 4, 6, 8 and 10, respectively.*

of phase-space region selected. It will be very interesting to see if the data show the same patterns when investigated with- and without extra cuts in multiplicity.

4 Higher moments for hadrons and for jets

As noted in the introduction, the existence of minimum (with negative values) in the q-dependence of moments H_q was related to the higher order corrections in perturbative QCD. It is obvious, however, that the perturbative QCD calculations yield the multiplicity distributions for gluons, and not for single hadrons. The identifications of higher moments for those two distributions means a rather bold extension of the assumption of parton-hadron duality, usually applied only to the average values.

To estimate the reliability of such an extension we applied to the generated events the default clustering algorithm of PYTHIA (PYCLUS) and investigated the multiplicity distributions of reconstructed jets (which may be expected to correspond to the gluon distribution more closely, than the distribution of single hadrons). The average number of jets depends strongly on the values of two parameters. One of them defines the maximal phase-space distance between the particles added to the existing jet; the other one defines the maximal CM momentum for the slowest particles in CM, which form a separate cluster/jet.

We calculated the ten lowest moments for the distributions of jets defined with those parameters spanning a range of $0.02 - 0.1$ GeV, for the L3 parameters, with a cut in the multiplicity. An additional degree of freedom was introduced by including in the clustering algorithm not only the charged stable hadrons (stable in the experimental sense, i.e. coming to the detectors; this is the condition used in all the distributions), but also stable neutral particles. These are mainly photons (coming from the π^0 decays), but also (anti)neutrons and long-living neutral kaons. This option allows to compare the distribution of hadrons and jets with the same average multiplicity. The results of our

calculations are shown in Fig.4. It is obvious that the values of moments for jets are very different from those for hadrons, even for relatively small jets (when the multiplicity in both cases is similar); e.g., the fourth moment is now negative.

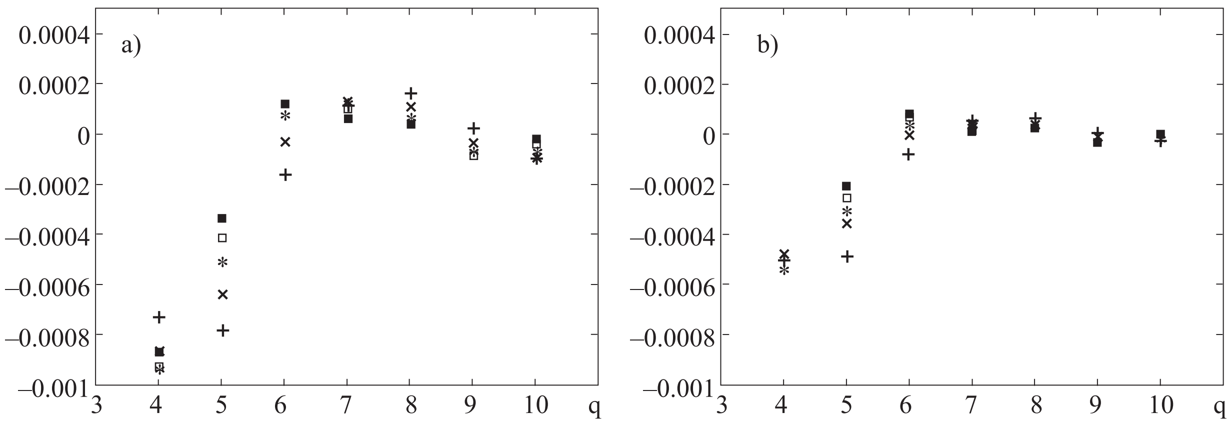

Figure 4: *The H_q moments for jets reconstructed from hadrons for PYTHIA events generated with L3 parameters. In Fig. (a) only charged hadrons are used; in Fig. (b) all stable particles are counted. Crosses, x-s, stars, open and closed squares correspond to the increasing average number of particles in a jet (from about 1.1 to 1.5 in Fig. (a) and from 1.3 to 2 in Fig. (b).*

When we count all the stable particles, we may select the values of jet defining parameters for which their average multiplicity is the same as the average number of charged stable hadrons. Also in this case we observe a striking difference between the values of higher moments for jets and hadrons. With decreasing average jet multiplicity (and increasing average number of particles in a single jet) even the second cumulant (not shown) becomes negative and all the pattern of the dependence of moments H_q on their order q does not resemble even roughly the pattern found for the single charged stable hadrons.

An experimental investigation of the H_q moments for jets has been performed by the L3 collaboration. The results are available in the D.J. Mangeol Ph.D. Thesis [8]. Although the jet definition used (Durham algorithm) was different than the default algorithm from PYTHIA used by us, the results are very similar to those presented in Fig. 4a: with increasing "jet size" the pattern of moments changes significantly.

We regard this dependence as a suggestion that the extended local parton-hadron duality (ELPHD), in which one identifies the (charged) hadron multiplicity distribution with the parton distribution at small fixed virtuality, is not very reliable. We should note, however, that the opposite conclusion was drawn from the same data [16]. The dependence of the H_q moments on their order q was investigated there for the parton jets in a MC model for varying jet resolution parameter Q_c. A good description of L3 data was found for jets as well as for hadrons. This was regarded as a support for ELPHD.

The agreement with the L3 data is very good indeed for $Q_c \geq 1GeV$. For small Q_c the agreement is reasonable, although not perfect. In particular, the ELPHD prediction for hadrons is practically identical to that for jets with $Q_c = 100MeV$, whereas in data there is a marked difference between the moments. This difference is similar to that seen for our PYTHIA events: a clear minimum for $q = 5$ seen for hadrons (Fig. 2) moves already

for smallest jets (Fig.4a) so that the values of H_4 and H_5 are almost equal. Thus there seems to be no clear "hadron limit" for parton MC, although the differences are not big.

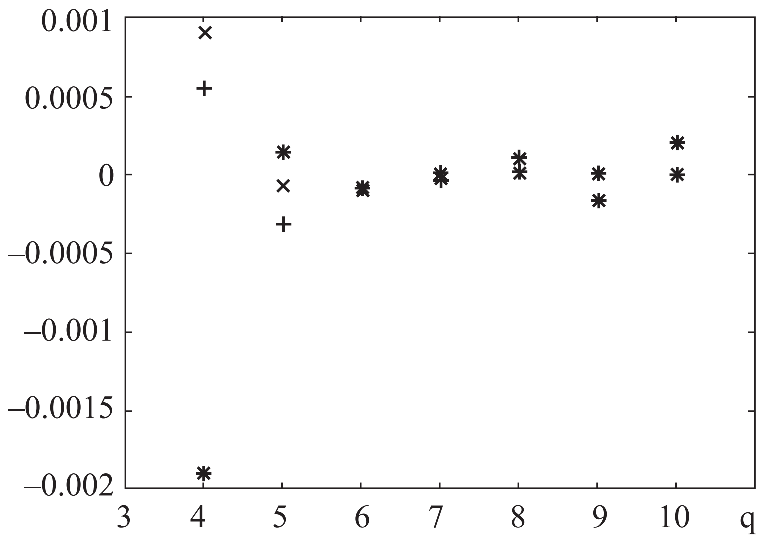

Figure 5: *The H_q moments for PYTHIA events generated with L3 parameters. Crosses denote the results for charged hadrons, x-s are for all stable particles and stars for positive hadrons.*

To verify further the possibility of describing hadron data by ELPHD we have compared the moments calculated for charged hadrons (pions, kaons, protons and antiprotons) with the moments calculated for all stable particles (thus adding photons, neutral kaons, neutrons and antineutrons). The results, shown in Fig.5, suggest that a relatively deep minimum at $q = 5$ is to a large extent due to the charge conservation effect. Indeed, the results for positive hadrons (shown also in Fig.5) do not exhibit a pattern seen for all charged hadrons. A deep minimum appears for $q = 4$, and the small oscillations with a very short period follow. In this case it is rather obvious that the data will confirm our findings, as the charge conservation is correctly built in PYTHIA and the charged-neutral correlations are known to be well described by this generator.

Obviously, there is no reason why the multiplicity distribution of partons should be reflected in the distribution of all charged hadrons more closely than in the distribution of all stable particles. Thus the observed difference between the moments calculated for these two choices seems to confirm our doubts concerning the ELPHD.

The choice of charged hadrons of both signs as a hadronic counterpart of partons (mostly gluons) in ELPHD is in fact rather surprising. The measured cross section for odd multiplicities of charged hadrons should be zero for an ideal detector, whereas the distribution of gluons should be smooth without any discrimination of odd multiplicity values. This suggests strongly that the agreement of moments calculated for partons and charged hadrons is to a large extent accidental and is due mostly to the similar influence of cuts and the two-component mechanism (mentioned in section 2) in both distributions. In our opinion, one should rather try to find parameters, for which the parton MC would agree with the distribution of positive (or negative) hadrons.

5 Conclusions and outlook

Using the PYTHIA generator we have investigated the multiplicity distributions for hadrons coming from the Z decay. We have calculated the ratios of cumulants to factorial moments. The results for charged hadrons in full phase-space are compatible with the L3 data. We confirm the earlier findings of a minimum at $q = 5, 6$ when the moments H_q are considered as functions of their order q. We confirm also that the truncation of the multiplicity distribution causes a significant enhancement of this minimum. However, increasing the statistics of generated events beyond $2M$ events does not change visibly the results. This excludes the "natural" truncation due to the final statistics as the main source of the minimum.

We find that the moments calculated from the multiplicity distributions in the limited part of phase-space differ significantly from that for the full phase-space. In particular, the minimum in the dependence of H_q on q shifts to higher values of q for smaller range of momenta. However, introducing an universal truncation on the multplicity distributions for different ranges of momenta, we recover a stable position of the minimum. It will be interesting to see if this pattern of generated events will be confirmed in the real data.

We have also investigated the moments for the jets, defined as clusters of particles close in momentum space. We find that even for very small jets, containing in average only slightly more than one hadron, the pattern of moments is different from that for hadrons. The same is true if we use the neutral particles to form jets, for which the average multiplicity is the same, as for charged hadrons. Since the distributions of such jets may be expected to correspond more closely to the distributions of partons, one may wonder if it is meaningful to compare the moments measured for the observed hadrons to the moments calculated for partons. We feel that our results cast doubts on the possibility of such an extension of "local parton-hadron duality" beyond the average quantities (moments of order one). Again, more investigation on the real data would be highly desirable.

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References

[1] A. Białas and R. Peschanski, *Nucl. Phys.* **B273**, 703 (1985).

[2] I.M. Dremin *et al.*, *Phys. Lett.* **B336**, 119 (1994).

[3] I.M. Dremin, *Phys. Lett.* **B313**, 209 (1993).

[4] I.M. Dremin, *JETP Lett.* **58**, 945 (1993).

[5] R. Ugoccioni, A. Giovannini and S. Lupia, *Phys. Lett.* **D59**, 094020 (1999).

[6] T. Sjöstrand *et al.*, *Comp. Phys. Comm.* **135**, 238 (2001).

[7] P. Achard *et al.*, L3 Collaboration, *Phys. Lett.* **B577**, 109 (2003).

[8] D.J. Mangeol, Correlations in the Charged-Particle Multiplicity Distribution, Ph.D. thesis, e-print hep-ph/0110029.

[9] T. Sjöstrand, *Comp. Phys. Comm.* **82**, 74 (1994).

[10] G. Marchesini *et al.*, *Comp. Phys. Comm.* **67**, 465 (1992).

[11] I.M. Dremin, e-print hep-ph/0404092, proceedings of the Moriond meeting 2004.

[12] A similar model for hadron-hadron collisions was first suggested by A. Giovannini and R. Ugoccioni, *Phys. Rev.* **B336**, 119 (1994).

[13] G.J. Alner *et al.*, UA5 collaboration, *Phys. Lett.* **B160**, 193 (1985).

[14] L. Van Hove, *Physica* **147A**, 19 (1987).

[15] K. Fiałkowski, *Phys. Lett.* **B173**, 197 (1986).

[16] M.A. Buican, C. Förster and W. Ochs, *Eur. Phys. J.* **C31**, 57 (2003); W. Ochs, *Acta Phys. Pol.* **B35**, 429 (2004).